ACKNOWLEDGMENT

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NOTATION

d_p = particle diameter, L
$f(\ )$ = function of ( )
g = acceleration due to gravity, L/θ^2
G_{mf} = minimum fluidization flow rate, $M/L^2\theta$
N_{Ga} = Galileo number = $d_p^3\, \rho_f(\rho_s - \rho_f)g/\mu^2$
N_{Re} = particle Reynolds number = $(d_p\, \rho_f V/\mu)$
$(N_{Re})_{mf}$ = particle Reynolds number at onset of fluidization
N_{Ret} = particle Reynolds number at terminal falling velocity
V = superficial fluid velocity, L/θ
ρ_f = fluid density, M/L^3
ρ_s = particle density, M/L^3
μ = fluid viscosity, $M/L\theta$
ϵ_{mf} = minimum fluidization voidage
ϕ_s = $\dfrac{\text{surface area of sphere having the same volume as particle}}{\text{surface area of particle}}$

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Scale-Up of Residence Time Distributions

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Residence time distributions are valuable for understanding the performance of many continuous flow systems and for formulating mathematical models of such systems. The residence time distribution $Y(X)$ referred to in this communication is defined as: $Y(X)dX$ is the fraction of the inflowing (outflowing) stream which will spend (has spent) a time between X and $X + dX$ in the system. For ideal cases such as plug flow, perfectly stirred vessels, laminar flow, etc., the residence time distributions may be obtained analytically. However, in many cases of practical interest, it is necessary to determine the distributions experimentally. This study investigates the possibility of determining residence time distributions of process systems from experimental tests performed on suitably scaled laboratory models.

CRITERIA FOR MODELS

As early as 1953 the conditions under which residence time distributions of large systems can be predicted from

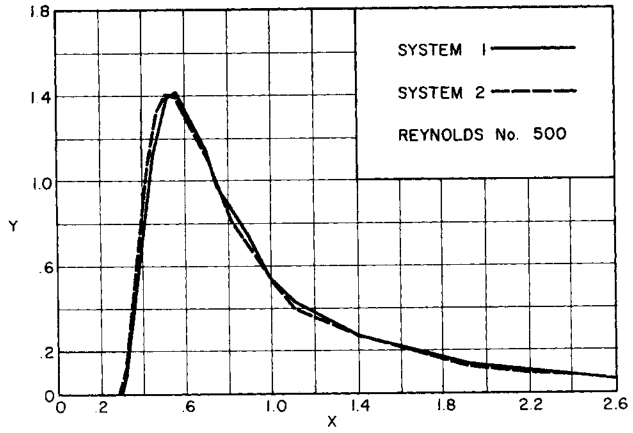

Fig. 1. Comparison of average residence time distributions.

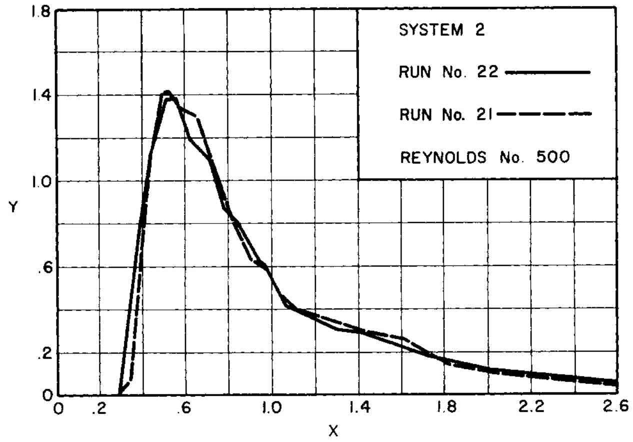

Fig. 2. Comparison of duplicate residence time distributions.

laboratory models were stated by Danckwerts (*1*) as follows: (1) The model must be geometrically similar to the system. (2) The Reynolds number must be the same in the model and the system. (3) Gravity waves, density differences, surface tension, and other influences apart from inertia and viscosity must be unimportant in determining the behavior of the fluid in both the model and system.

For the class of systems studied here, namely, tubular vessels in which an incompressible fluid with constant physical properties is flowing at a steady rate, condition (3) is satisfied. Hence the criteria for constructing and operating models are (1) geometrical similarity and (2) equal Reynolds number. An independent, detailed derivation based on homologous transformation of the Navier-Stokes equation confirms these criteria.*

EXPERIMENTAL

Two tubular vessels were constructed and operated according to the restrictions of the preceding section. Light-sensitive solution was forced by compressed nitrogen through a series circuit consisting of a flash box, tubular vessel, colorimeter, rotameter, and needle valve. The color of the outlet stream was continuously monitored by the colorimeter and recorded.

The tubular vessels (I.D. 1.125- and 2.25-in.) were constructed from transparent Plexiglas pipes of ¼-in. wall thickness with flat flanges at each end. The ratio of length to diameter was 8:1. The ratio of the diameter of the inlet and outlet tubes to the diameter of the main vessel was 1:5.5. The inlet and outlet tubes were also constructed from Plexiglas and were attached to the flanges of the tubular vessel in a concentric fashion.

In order to provide rapid injection of tracer and to avoid mechanical agitation which would disturb the entering flow patterns, a method of flash photolysis was used to introduce tracer (*2*). The pulse formed was nearly a true delta function; the volume of tracer was less than 0.1% of the volume of the vessel.

Twenty-seven experiments distributed over three Reynolds numbers were performed. Between three and six experiments were made under each set of conditions. The Reynolds numbers used were 400, 500, and 600 based on the diameter of the test vessel. In addition, supplementary experiments were performed at Reynolds numbers of 300 and 800 to obtain starting and peak times. Equipment limitations precluded a more detailed investigation of a wider range of operating conditions. The narrowness of the range of Reynolds numbers investigated was partially mitigated by the fact that the flow was not laminar, but rather complex due to end effects and the formation of a jet at the entrance.

* A more leisurely version of this paper containing detailed derivations and additional data has been prepared under the title "Scale-Up of Residence Time Distributions," and is available from the authors, Chemical Engineering Science Group, Case Institute of Technology, Cleveland, Ohio 44106.

RESULTS AND CONCLUSIONS

For all of the Reynolds numbers studied, it was found practical to predict the residence time distribution of the larger vessel from the smaller one (*3*). A typical set of data (Reynolds number = 500) is shown in Figure 1, which shows the close agreement obtained between the residence time distribution of the larger vessel (system 1) and the model (system 2). Each of the distributions shown in Figure 1 is the average of from three to six individual runs.

For comparison, the deviation between two typical duplicate runs made on the model (Reynolds number = 500) is shown in Figure 2. The deviation between duplicate runs is seen to be greater than the deviation between the averages of the larger system and its model. The variation between duplicate runs is not primarily due to experimental error, but rather is inherent because of the stoichastic nature of eddies. Typical variations between duplicate determinations of residence time distributions are of the order of 10% in peak height, peak time, and starting time. It is for this reason that residence time distributions have been averaged for comparison. A more detailed quantitative examination of these results, including a statistical analysis (*3*), confirms the above qualitative observations.

From these results, it is concluded that if the only criteria governing the construction and operation of models of a system are geometrical similarity and equal Reynolds numbers, then it is practical to determine the residence time distribution of a system from experiments on laboratory-size models. This study demonstrates the practicality of making such measurements and also illustrates the convenience of using flash photolysis for introducing tracer.

NOTATION

- Y = dimensionless concentration (CV/Q)
- X = dimensionless time (vt/V)
- C = concentration
- Q = quantity of tracer injected into system
- t = time
- v = volumetric flow rate
- V = volume of the system

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